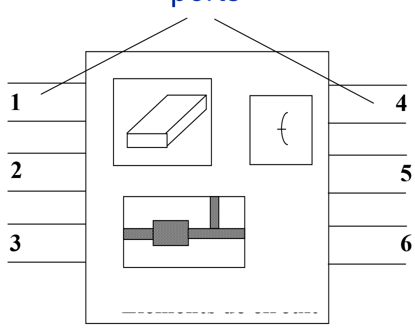
Microwave circuit characterization

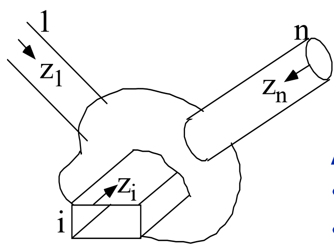
Microwave component

ports



Circuit elements

Reference planes



On each access line i of a component, a coordinate axis z_i is defined. The origin of this axis lies in the reference plane of the port i.

Assumptions :
•The transmission lines are lossless
•They support only the dominant mode
•The reference plane is distant enough From discontinuities to ensure that the Higher order modes are attenuated

Available models

- Kirchhoff
  - Currents and voltages ill defined
  - Well known from low frequency circuit analysis
  - Not adapted to system approach
- Scattering matrix
  - Based on signal and power
  - Well suited to microwaves
  - Not known from low frequency circuit analysis
  - Adapted to system analysis

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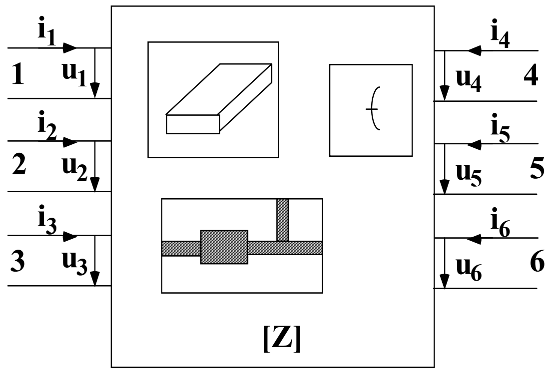
Kirchhoff's model



Current, voltages and impedances

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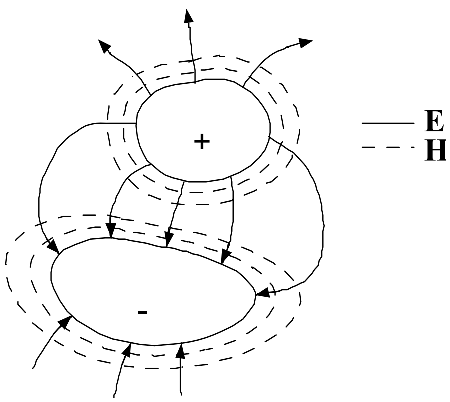
Current voltages and impedances

- Easy to use
- Current and voltage difficult to measure
- Current and voltage not always uniquely defined

Example : TEM line



$$\mathbf{E} = -\nabla V$$

$$V = \int_{+}^{-} \mathbf{E} \cdot \mathbf{dl}$$

$$I = \oint_{C+} \mathbf{H} \cdot \mathbf{dl}$$

Characteristic impedance

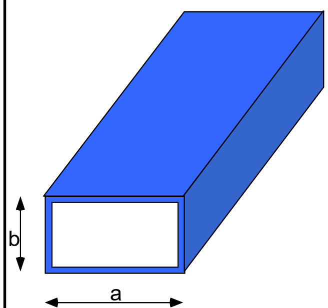
$$Z_c = \frac{V}{I} = \sqrt{\frac{L}{C}}$$

Example : TEM line

- Univoque definition of voltage and current
- Fields identical to static fields
- Zero cutoff frequency
- Needs at least two separate conductors

Non-TEM line : the rectangular waveguide

$$\mathbf{E} = -\nabla V - j\omega\mu\mathbf{A}$$



Example : mode TE10 of a rectangular waveguide

$$E_y(x,y,z) = E_0 \frac{j\omega\mu a}{\pi} \sin\frac{\pi x}{a} e^{-j\beta z} = E_0 e_y(x,y) e^{-j\beta z}$$

$$H_x(x,y,z) = E_0 \frac{j\beta a}{\pi} \sin\frac{\pi x}{a} e^{-j\beta z} = E_0 h_y(x,y) e^{-j\beta z}$$

Thus

$$V = E_0 \frac{-j\omega\mu a}{\pi} \sin\frac{\pi x}{a} e^{-j\beta z} \int_y dy$$

Non TEM voltage and current definition

- Only defined for one mode
- The voltage is proportional to the transverse electric field
- The current is proportional to the transverse magnetic field
- The characteristic impedance is equal to U/I. We choose the characteristic impedance equal to the wave impedance
- The power flux is given by the product of the voltage and the current

Arbitrary line

$$\mathbf{E_t}(x,y,z)=\mathbf{e_t}(x,y)\left(E_o^+e^{-j\beta z}+E_o^-e^{j\beta z}\right)$$
$$=\frac{\mathbf{e_t}(x,y)}{C_1}\left(V^+e^{-j\beta z}+V^-e^{j\beta z}\right)$$
$$\mathbf{H_t}(x,y,z)=\mathbf{h_t}(x,y)\left(E_o^+e^{-j\beta z}-E_o^-e^{j\beta z}\right)$$
$$=\frac{\mathbf{h_t}(x,y)}{C_2}\left(I^+e^{-j\beta z}-I^-e^{j\beta z}\right)$$

$$V(z)=V^+e^{-j\beta z}+V^-e^{j\beta z}$$
$$I(z)=I^+e^{-j\beta z}-I^-e^{j\beta z}$$

$$Z_c=\frac{V^+}{I^+}=\frac{V^-}{I^-}=\frac{C_1E_o^+}{C_2E_o^+}=\frac{C_1}{C_2}$$

$$\frac{C_1}{C_2}=Z_{\mathrm{mod}}$$

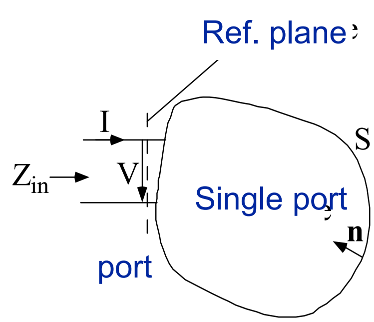
Different impedances

- Impedance of the medium : $Z_o=\sqrt{\frac{\mu}{\varepsilon}}$
- Wave impedance : $Z_{\mathrm{mod}}=\frac{|\mathbf{E_t}|}{|\mathbf{H_t}|}$
- Line characteristic impedance : $Z_c=\frac{V^+}{I^+}=\frac{V^-}{I^-}=\sqrt{\frac{L}{C}}$
- Impedance matrix of a circuit

Impedance matrix

Single port element

$$Z_{in} = \frac{V}{I}$$



Power delivered to the component (Poynting)

$$P = \oint_s \boldsymbol{E} \times \boldsymbol{H}^* \cdot \boldsymbol{ds} = P_r + 2j\omega(W_m - W_e)$$

Single port element

Voltage and current :

$$\mathbf{E_t}(x,y,z)=V(z)\frac{\mathbf{e_t}(x,y)}{C_1}e^{-j\beta z}$$

$$\mathbf{H_t}(x,y,z)=I(z)\frac{\mathbf{h_t}(x,y)}{C_2}e^{-j\beta z}$$

with $\frac{1}{C_1C_2}\int_s \mathbf{e_t}\times\mathbf{h_t}\cdot\mathbf{ds}=1$ thus $P=\frac{1}{C_1C_2}\int_s VI^*\boldsymbol{e_t}\times\boldsymbol{h_t}\cdot\boldsymbol{ds}=VI^*$

and

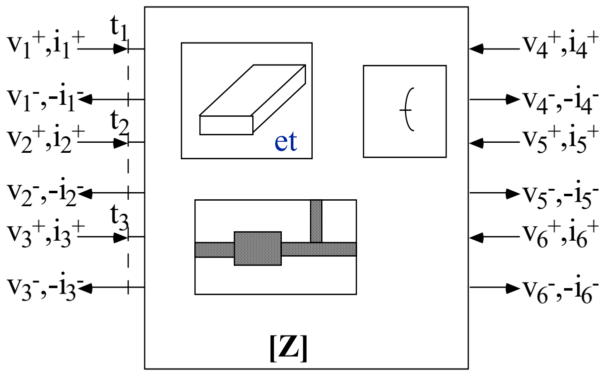
$$Z_{in}=R+jX=\frac{V}{I}=\frac{VI^*}{|I|^2}=\frac{P}{|I|^2}$$

$$=\frac{\left(P_r+2j\omega\left(W_m-W_e\right)\right)}{|I|^2}$$

Single port element

- R is proportional to to the real power dissipated in the system (Losses)
- X is proportional to the mean reactive stored energy in the system

Impedance and admittance matrix



$$V_n = V_n^+ + V_n^-$$

$$I_n = I_n^+ - I_n^-$$

At each port t_i

Impedance and admittance matrix

$$[V] = [Z][I]$$
$$[I] = [Y][V]$$

$$Z_{ij} = \left.\frac{V_i}{I_j}\right|_{I_k=0 \quad pour \quad k\neq j}$$

$$Y_{ij} = \left.\frac{I_i}{V_j}\right|_{V_k=0 \quad pour \quad k\neq j}$$

- Impedance matrix is obtained in open circuit conditions
- Admittance matrix is obtained in short circuit conditions

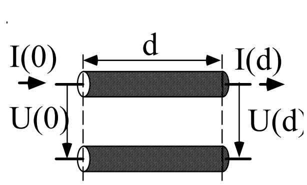
Properties of the impedance and admittance matrix

- Reciprocity : $Y_{ij} = Y_{ji}$

  $Z_{ij} = Z_{ji}$

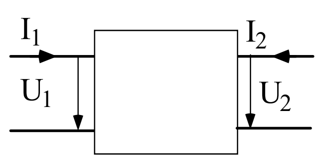
- Lossless circuit : $\mathrm{Re}\{Z_{mn}\} = 0$

Example : transmission line



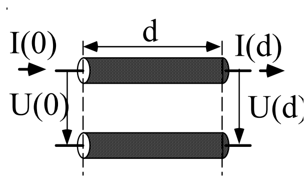
$$U_1 = U(0), I_1 = I(0),$$
$$U_2 = U(d), I2 = -I(d)$$

$$U(z) = U_+ \,\mathrm{e}^{-\gamma z} + U_- \,\mathrm{e}^{+\gamma z}$$
$$I(z) = I_+ \,\mathrm{e}^{-\gamma z} + I_- \,\mathrm{e}^{+\gamma z}$$

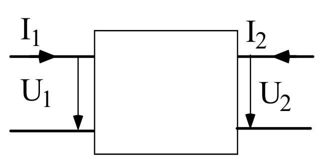


$$U(0) = U_+ + U_- \qquad U(d) = U_+ \,\mathrm{e}^{-\gamma d} + U_- \,\mathrm{e}^{+\gamma d}$$
$$I(0) = I_+ + I_- \qquad I(d) = I_+ \,\mathrm{e}^{-\gamma d} + I_- \,\mathrm{e}^{+\gamma d}$$

Example : transmission line



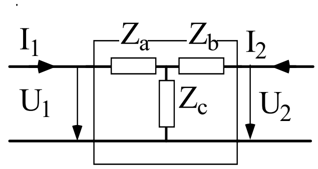
$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = Z_c \begin{bmatrix} \coth(\gamma d) & \dfrac{1}{\sinh(\gamma d)} \\ \dfrac{1}{\sinh(\gamma d)} & \coth(\gamma d) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = Y_c \begin{bmatrix} \coth(\gamma d) & \dfrac{-1}{\sinh(\gamma d)} \\ \dfrac{-1}{\sinh(\gamma d)} & \coth(\gamma d) \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

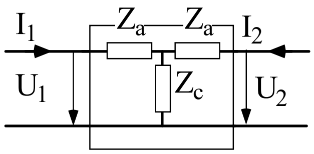
Equivalent T circuit of a reciprocal two-port

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12} & Z_{22} \end{bmatrix}$$



$$Z_a = Z_{11} - Z_{12}$$
$$Z_b = Z_{22} - Z_{12}$$
$$Z_c = Z_{12}$$

Equivalent T circuit of a transmisson line



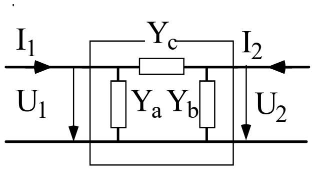
$$Z_a = Z_{caract}\left(\coth(\gamma d) - \frac{1}{\sinh(\gamma d)}\right)$$

$$= Z_{caract}\left(\frac{ch(\gamma d)-1}{\sinh(\gamma d)}\right) = Z_{caract}\tanh\left(\frac{\gamma d}{2}\right)$$

$$Z_c = \frac{Z_{caract}}{\sinh(\gamma d)}$$

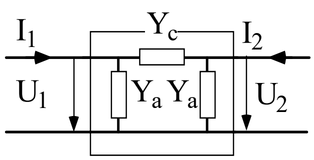
Equivalent Π circuit of a reciprocal two-port

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12} & Y_{22} \end{bmatrix}$$



$$Y_a = Y_{11} + Y_{12}$$
$$Y_b = Y_{22} + Y_{12}$$
$$Y_c = -Y_{12}$$

Equivalent Π circuit of a transmisson line



$$Y_a = Y_{caract}\left(\coth(\gamma d) - \frac{1}{\sinh(\gamma d)}\right)$$

$$= Y_{caract}\left(\frac{ch(\gamma d)-1}{\sinh(\gamma d)}\right) = Y_{caract}\tanh\left(\frac{\gamma d}{2}\right)$$

$$Y_c = \frac{Y_{caract}}{\sinh(\gamma z)}$$

Scattering parameters

Normalized wave amplitudes : definition

Normalized wave amplitudes

$$a_i = \frac{v_i + Z_{ci} i_i}{2\sqrt{Z_{ci}}} \quad , \quad b_i = \frac{v_i - Z_{ci} i_i}{2\sqrt{Z_{ci}}} \qquad [W^{1/2}]$$

Inverse relation

$$v_i = \sqrt{Z_{ci}}\left(a_i + b_i\right) \quad , \quad i_i = \frac{\left(a_i - b_i\right)}{\sqrt{Z_{ci}}}$$

Normalized wave amplitudes : properties

Transmission lines

$$v_i = v_i^+ e^{-j\beta z} + v_i^- e^{+j\beta z}$$

$$i_i = i_i^+ e^{-j\beta z} + i_i^- e^{+j\beta z}$$

$\longrightarrow$

$$a_i = \frac{v_i^+}{\sqrt{Z_{ci}}} e^{-j\beta z}$$

$$b_i = \frac{v_i^-}{\sqrt{Z_{ci}}} e^{+j\beta z}$$

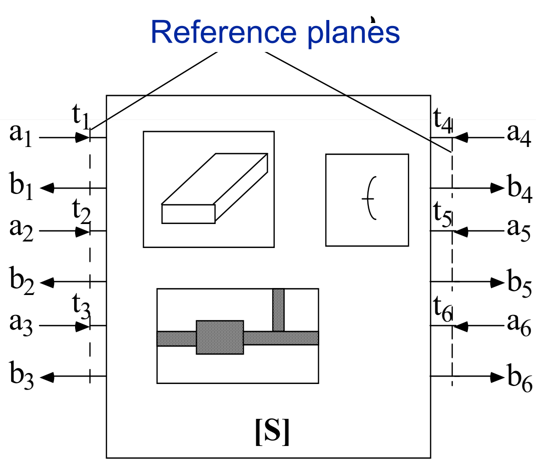
a_i : progressive wave
b_i : retrograde wave

Power at one component port

$$P_i = \mathrm{Re}\left[v_i i_i^*\right] = \mathrm{Re}\left[(a_i + b_i)(a_i^* - b_i^*)\right] = |a_i|^2 - |b_i|^2$$

$/a_i/^2$: power entering port
$/b_i/^2$: power leaving port

Signal model



$$[b] = [S][a]$$

$$s_{ij} = \left.\frac{b_i}{a_j}\right|_{a_k = 0\ ,\ k \neq j}$$

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Scattering matrix : definition

Normalized wave amplitude

$$a_i = \frac{v_i + Z_{ci} i_i}{2\sqrt{Z_{ci}}} \quad , \quad b_i = \frac{v_i - Z_{ci} i_i}{2\sqrt{Z_{ci}}}$$

Inverse relation

$$v_i = \sqrt{Z_{ci}}\left(a_i + b_i\right) \quad , \quad i_i = \frac{\left(a_i - b_i\right)}{\sqrt{Z_{ci}}}$$

Scattering matrix

$$[b] = [S][a] \qquad s_{ij} = \left.\frac{b_i}{a_j}\right|_{a_k = 0 \ , \ k \neq j}$$

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Signal model

- Advantages :
  - Signals are easy to measure
  - Well suited to a system approach (transfer functions)
- Drawbacks :
  - Not known at low frequencies

Scattering matrix : properties

- Sij : transfert function from port j to port i
- Depends on the component and its properties
- To change the connecting lines implies to chnage the scattering matrix
- The scattering matrix is obtained termiating the ports by matched loads

Reciprocity

Passive, linear, isotropic circuit

↓

The circuit is reciprocal

↓

$$z_{ij} = z_{ji}$$

↓

$$s_{ij} = s_{ji}$$

Lossless circuit

$$\sum |a_i|^2 = \sum |b_i|^2 \longrightarrow [\tilde{a}][a] - [\tilde{b}][b] = 0 \quad \text{With } [\tilde{a}] = [a^*]^t$$

Moreover $[b] = [S][a]$, $[\tilde{b}] = [\tilde{a}][\tilde{S}]$ $\longrightarrow$

$$[\tilde{a}][a] - [\tilde{a}][\tilde{S}][S][a] = 0$$

$$[\tilde{a}]\{[1] - [\tilde{S}][S]\}[a] = 0 \longrightarrow$$

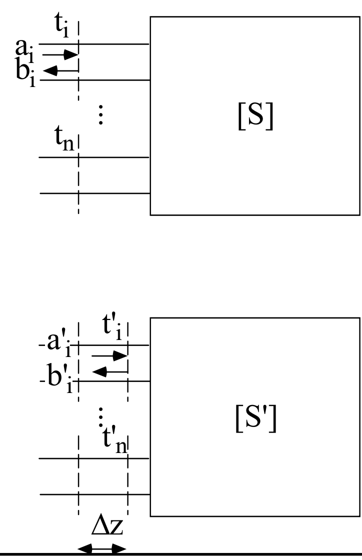
$$[\tilde{S}][S] = [1]$$

$$\sum_{i=1}^{N} s_{ij}^* s_{ik} = \delta_{jk} \qquad \delta_{jk} = \begin{cases} 1 & si \quad j = k \\ 0 & si \quad j \neq k \end{cases}$$

Matched circuit

Sii = 0

Change of reference plane



Change of reference plane

$$a'_i = a_i e^{-j\varphi_i}$$

$$b'_i = b_i e^{j\varphi_i}$$

$$\varphi_i = -\beta_i \Delta z_i$$

$$[a] = \left[diag\ \ e^{j\varphi}\right][a']$$

$$[a'] = \left[diag\ \ e^{-j\varphi}\right][a]$$

$$[b] = \left[diag\ \ e^{-j\varphi}\right][b']$$

$$[b'] = \left[diag\ \ e^{j\varphi}\right][b]$$

thus $s'_{ii} = s_{ii} e^{j2\varphi_i}$

And in general

$$\left[diag\ \ e^{j\varphi}\right] = \begin{bmatrix} e^{j\varphi_1} & 0 & \dots & 0 \\ 0 & e^{j\varphi_2} & & : \\ : & & & \\ 0 & \dots & & e^{j\varphi n} \end{bmatrix}$$

Change of reference plane

$$[b'] = \left[diag\ \mathrm{e}^{j\varphi}\right][S]\left[diag\ \mathrm{e}^{j\varphi}\right][a']$$

$$[S'] = \left[diag\ \mathrm{e}^{j\varphi}\right][S]\left[diag\ \mathrm{e}^{j\varphi}\right]$$

$$s'_{ij} = s_{ij}\ \mathrm{e}^{j\left(\varphi_i + \varphi_j\right)}$$

Passive circuits : questions

1. Linearity
2. Reciprocity
3. Losslessness
4. Matched
5. Symmetry

[S]->[Z]

$[\underline{b}] = [\underline{S}]\,[\underline{a}]$ $\quad \underline{a}_i = (\underline{U}_i + Z_{ci}\,\underline{I}_i)/2\sqrt{Z_{ci}}$ $\quad \underline{U}_i = \sqrt{Z_{ci}}\,(\underline{a}_i + \underline{b}_i)$

$[\underline{U}] = [\underline{Z}]\,[\underline{I}]$ $\quad \underline{b}_i = (\underline{U}_i - Z_{ci}\,\underline{I}_i)/2\sqrt{Z_{ci}}$ $\quad \underline{I}_i = (\underline{a}_i - \underline{b}_i)/\sqrt{Z_{ci}}$

We define two auxiliary matrices

$[G] = [\mathrm{diag}(Z_{ci})]$ $\qquad [F] = [\mathrm{diag}(1/2\sqrt{Z_{ci}})]$

And we can write

$[\underline{a}] = [F]\,\{[\underline{U}] + [G]\,[\underline{I}]\}$ $\qquad ([\underline{a}] + [\underline{b}]) = 2\,[F]\,[\underline{U}]$

$[\underline{b}] = [F]\,\{[\underline{U}] - [G]\,[\underline{I}]\}$ $\qquad ([\underline{a}] - [\underline{b}]) = 2\,[F]\,[G]\,[\underline{I}]$

[S]->[Z]

$[\underline{a}] = [F]\,\{[\underline{U}] + [G]\,[\underline{I}]\} \rightarrow [\underline{a}] = [F]\,\{[\underline{Z}] + [G]\}\,[\underline{I}]$

$\rightarrow [\underline{I}] = \{[\underline{Z}] + [G]\}^{-1}\,[F]^{-1}\,[\underline{a}]$

$[\underline{b}] = [F]\,\{[\underline{U}] - [G]\,[\underline{I}]\} \rightarrow [\underline{b}] = [F]\,\{[\underline{Z}] - [G]\}\,[\underline{I}]$

$\rightarrow [\underline{b}] = [F]\,\{[\underline{Z}] - [G]\}\,\{[\underline{Z}] + [G]\}^{-1}\,[F]^{-1}\,[\underline{a}] = [\underline{S}]\,[\underline{a}]$

Thus: $[\underline{S}] = [F]\,\{[\underline{Z}] - [G]\}\,\{[\underline{Z}] + [G]\}^{-1}\,[F]^{-1}$

In one dimension, we have $\underline{\rho} = \underline{S}_{11} = (\underline{Z}_t - Z_c)/(\underline{Z}_t + Z_c)$

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[Z]->[S]

$([\underline{a}] + [\underline{b}]) = 2\,[F]\,[\underline{U}] \;\mathbf{->}\; [\underline{U}] = (2\,[F])^{-1}([\underline{a}] + [\underline{b}]) = (2\,[F])^{-1}([1] + [\underline{S}])\,[\underline{a}]$

$([\underline{a}] - [\underline{b}]) = 2\,[F]\,[G]\,[\underline{I}] = ([1] - [\underline{S}])\,[\underline{a}] \;\mathbf{->}\; [\underline{a}] = ([1] - [\underline{S}])^{-1}\,2\,[F]\,[G]\,[\underline{I}]$

regrouping, we obtain:

$$[\underline{U}] = (2\,[F])^{-1}([1] + [\underline{S}])\,([1] - [\underline{S}])^{-1}\,2\,[F]\,[G]\,[\underline{I}] = [\underline{Z}]\,[\underline{I}]$$

Et on en tire: $[\underline{Z}] = ([F])^{-1}([1] + [\underline{S}])\,([1] - [\underline{S}])^{-1}\,[F]\,[G]$

For a single acess component, we have

$$\underline{Z}t = Zc(1 + \underline{r})/(1 - \underline{r})$$

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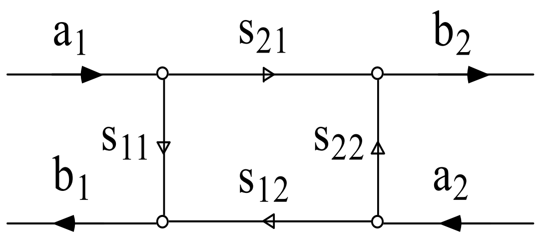
Flow charts

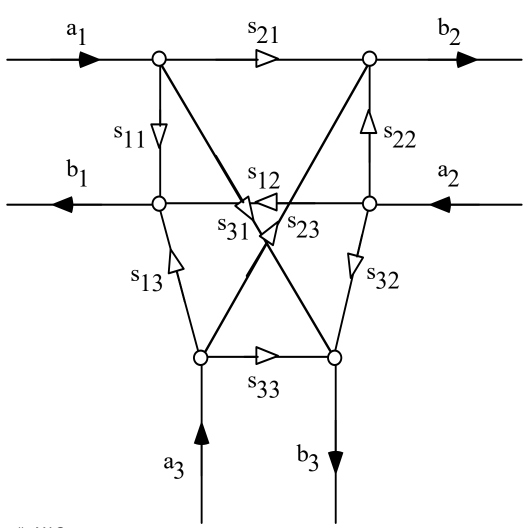
The scattering parameters (reflection and transfer functions) are represented as arrows

Flow charts



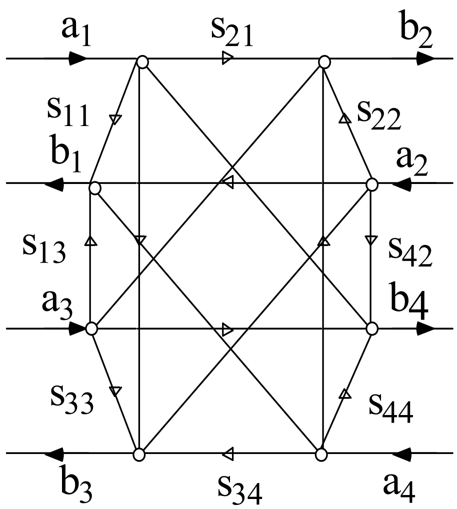
Two port

Flow charts



Three port

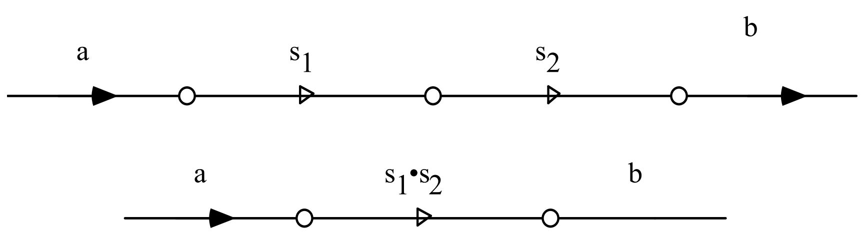
Flow charts



4-port

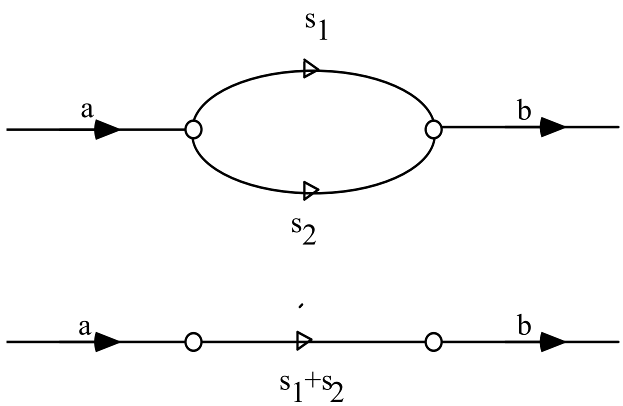
Flow chart reduction rules

1) multiplication



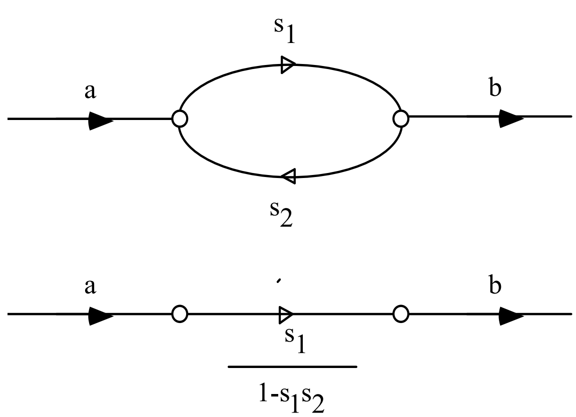
Flow chart reduction rules

2) addition

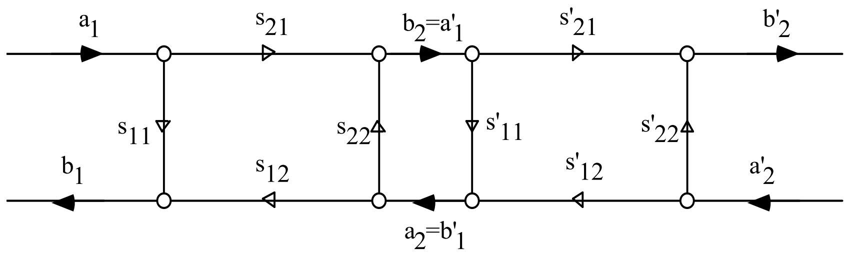


Flow chart reduction rules

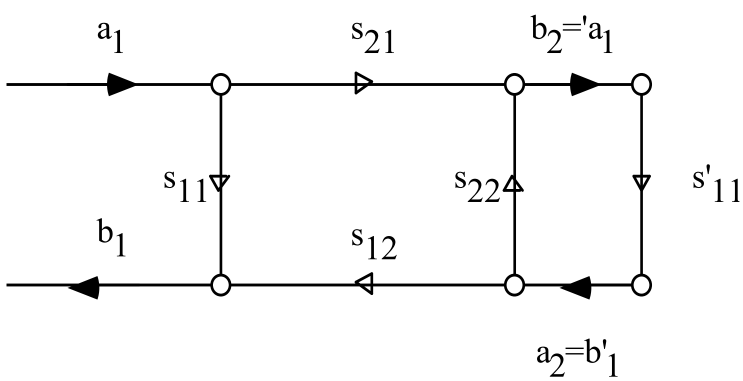
3) retroaction



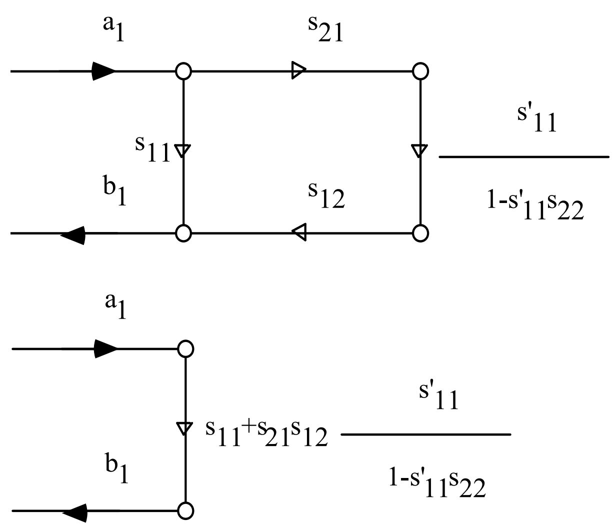
Example



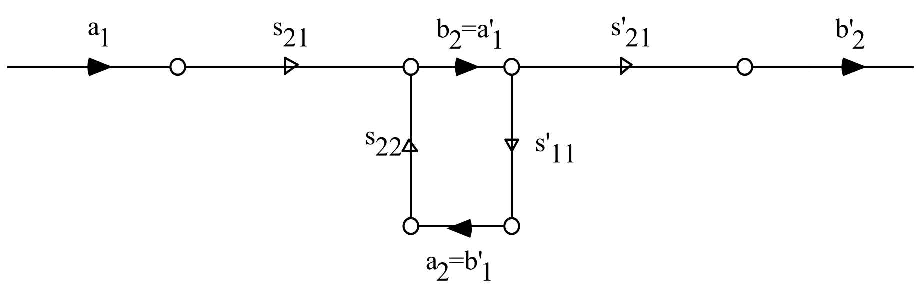
Example



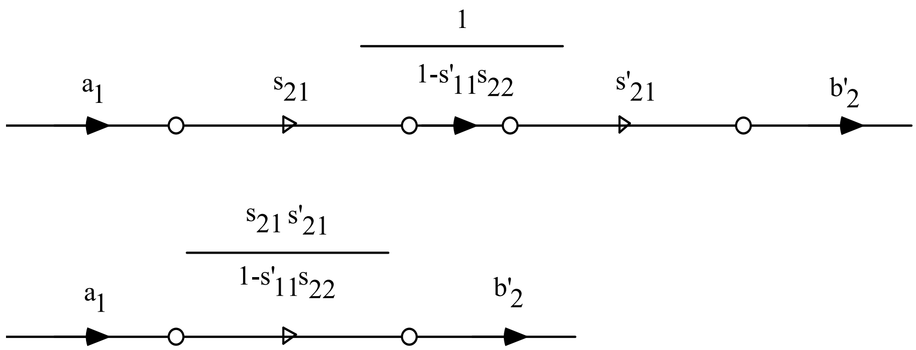
Example



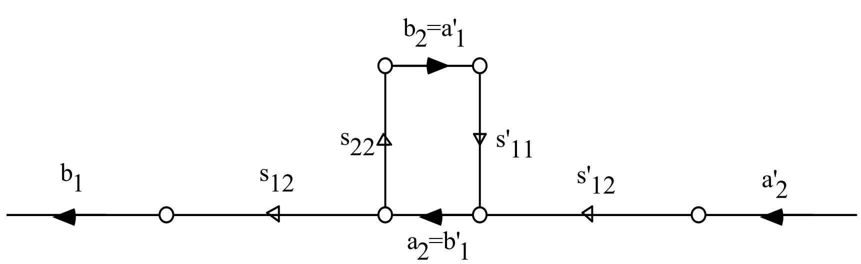
Example



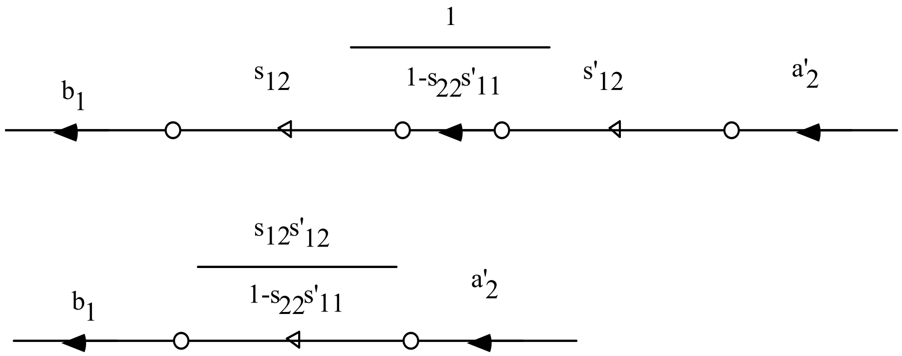
Example



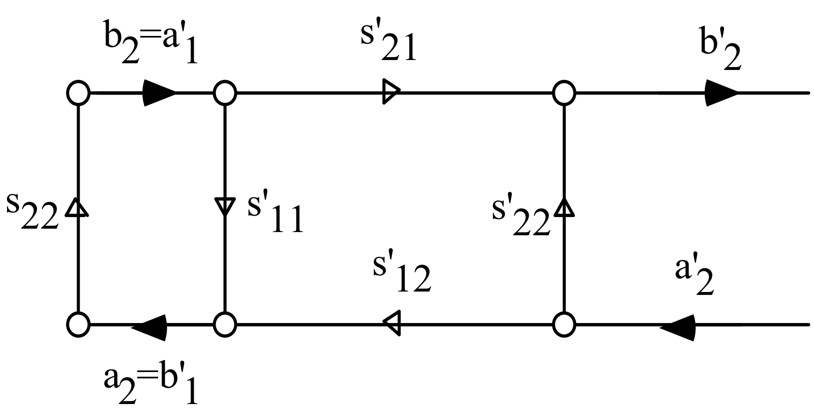
Example



Example



Example



Example